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Reflection of light from nonasbsorbing semi-infinite cloudy media: a simple approximation

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Abstract

The paper is devoted to the derivation of a simple approximate equation for the reflection function of semi-infinite cloudy media for the case of small and moderate observation angles. The absorption of light in cloud is neglected. The accuracy of equation obtained is studied using the numerical solution of the radiative transfer equation. It was found that approximation derived can be used for large scattering angles (larger than 150°) with accuracy better than 5% if observation angles are smaller than 45°. The error is below 5% for observation angles smaller than 30°, incidence angles smaller than 55° and arbitrary relative azimuths of incident and reflected light beams.

Keywords: Radiative transfer theory; Light reflection; Semi-infinite media

1. Introduction

The reflection function of a semi-infinite nonabsorbing medium plays an important role in the general radiative transfer theory [1]. In particular, its knowledge allows for the calculation of transmission, reflection, and absorption characteristics of light scattering layers of large but finite thickness (e.g., optical thickness $\tau \geqslant 10$ for clouds), using asymptotic radiative transfer equations [2,3]. This is why this function has been studied extensively by many authors [1,4–8]. In particular, a user-friendly and highly accurate code for the calculation of this function is available on the Internet [6]. Various approximate solutions for this function also has been reported [1,5,7,8]. The task of this paper is to derive a simple and yet accurate approximation for the reflection function in a special case of water

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clouds in visible. In essence this is the extension of our approach reported earlier [8] on the case of non-nadir measurements. We propose a simple parameterization for this function and check its accuracy. The approach proposed can be easily extended to the case of discrete random media other than clouds.

2. Theory

The reflection function of a semi-infinite layer $R(\zeta,\eta,\varphi)$ (ζ is the cosine of the observation angle ϑ_1, η is the cosine of the incidence angle ϑ_0, and φ is the relative azimuth of incident and reflected beams) can be written as [6]

$$R(\zeta,\eta,\varphi) = \bar{R}(\zeta,\eta) + 2\sum_{m=1}^{\infty} R_m(\zeta,\eta)\cos(m\varphi). \tag{1}$$

Here

$$\bar{R}(\zeta,\eta) = \frac{1}{2\pi}\int_0^{2\pi} R(\zeta,\eta,\varphi)\,\mathrm{d}\varphi \tag{2}$$

and

$$R_m(\zeta,\eta) = \frac{1}{2\pi}\int_0^{2\pi} R(\zeta,\eta,\varphi)\cos(m\varphi)\,\mathrm{d}\varphi. \tag{3}$$

$\bar{R}(\zeta,\eta)$ gives the reflection function averaged over the azimuth and $R_m(\zeta,\eta)$ is the mth Fourier coefficient of the reflection function. Fourier components $R_m(\zeta,\eta)$ for water clouds were studied in detail by King [9].

Eq. (1) is a very convenient starting point for the parameterization purposes because it decouples (ζ,η)-dependences from the azimuthal dependence φ. It is much easier to parameterise functions $\bar{R}(\zeta,\eta) = \bar{R}(\eta,\zeta)$ and $R_m(\zeta,\eta) = R_m(\eta,\zeta)$ than the three-parameter function $R(\zeta,\eta,\varphi)$ [7].

The second term in Eq. (1) is of no importance for nadir observations, where the azimuth φ plays no role due to the symmetry of the problem [8]. For non-nadir (but not too close to a horizon) observations the single light scattering gives a major contribution to the second term in Eq. (1). This contribution can be easily found [5]. It is given by the following equation:

$$S = \frac{p(\theta) - \bar{p}(\theta)}{4(\zeta+\eta)}, \tag{4}$$

where

$$\bar{p}(\theta) = \frac{1}{2\pi}\int_0^{2\pi} p(\theta)\,\mathrm{d}\varphi, \tag{5}$$

$$\theta = \arccos(-\zeta\eta + \sqrt{(1-\zeta^2)(1-\eta^2)}\cos\varphi) \tag{6}$$

and $p(\theta)$ is so-called phase function. This function gives a conditional probability of light scattering in a fixed direction and normalized as follows:

$$\frac{1}{2}\int_{-1}^{1} p(\theta)\sin\theta\,\mathrm{d}\theta = 1, \tag{7}$$

where θ is the scattering angle.

Replacing the second term in Eq. (1) by Eq. (4) we obtain [5]:

$$R(\zeta,\eta,\varphi)=\bar{R}(\zeta,\eta)+\frac{p(\theta)-\bar{p}(\theta)}{4(\zeta+\eta)}. \tag{8}$$

Note that the term

$$R_{\mathrm{ss}}=\frac{p(\theta)}{4(\zeta+\eta)} \tag{9}$$

gives the contribution of single scattering into reflection function of a semi-infinite layer [3,10].

In essence, Eq. (8) states that only the azimuth dependence of singly scattered light plays a primary role in the overall $R(\varphi)$-dependence. This is generally not a correct statement as shown, e.g., by King [9] and Melnikova et al. [7]. However, such an approach can be used with a high accuracy in a number of viewing geometries as specified below.

Clearly, the azimuth dependence plays no role for the nadir observation ($\zeta=1$) and illumination ($\eta=1$) conditions. Then we have

$$R(1,\eta)=R(\zeta,1)=\bar{R}(1,\eta)=\bar{R}(\zeta,1). \tag{10}$$

This means that the second term in Eq. (8) should vanish in this case. This is indeed the case. In particular, we have from Eq. (6) at $\eta=1$: $\theta=\pi-\vartheta_1$ independently on the azimuth and, therefore, $\bar{p}(\theta)=p(\theta)$ (see Eq. (5)).

The parameterization of the function $\bar{R}(\zeta,\eta)$ can be achieved as follows. First of all we note that it follows for isotropic scattering ($p(\theta)\equiv 1$) [10]:

$$R(\zeta,\eta)=\bar{R}(\zeta,\eta)=\frac{\varphi(\zeta)\varphi(\eta)}{4(\zeta+\eta)}, \tag{11}$$

where the function $\varphi(\zeta)$ can be approximated by the following linear dependence [3]:

$$\varphi(\zeta)=\alpha(1+2\zeta). \tag{12}$$

where $\alpha=4\sqrt{3}/7$. Substitution of Eq. (12) into Eq. (11) gives

$$\bar{R}(\zeta,\eta)=\frac{A+B(\zeta+\eta)+C\zeta\eta}{4(\zeta+\eta)}, \tag{13}$$

where $A=\frac{48}{49}\approx 1, B=2A, C=4A$.

Now we assume that Eq. (13) holds also for the case $p(\theta)\neq 1$ but coefficients A,B,C then depend on the phase function of a particular medium. In the first approximation only the dependence of A,B, and C on the asymmetry parameter

$$g=\frac{1}{2}\int_0^{\pi} p(\theta)\sin\theta\cos\theta\,\mathrm{d}\theta \tag{14}$$

may be considered as it was done, e.g., by Melnikova et al. [7] for a particular case of a Henyey–Greenstein phase function. Note that the parameter g for water clouds, considered here, varies only weakly in visible ($g\approx 0.85$) and we can assume the same constants A,B,C for clouds with arbitrary microphysics.

The approximate validity of Eq. (13) is confirmed by the numerical calculations using the Ambartsumian's nonlinear integral equation for $R(\zeta,\eta,\varphi)$. These results (for the function $D(\zeta,\eta)\equiv 4(\zeta+\eta)\bar{R}(\zeta,\eta)$) are given in Fig. 1. Indeed we see that the dependence $D(\eta)$ is very close to a

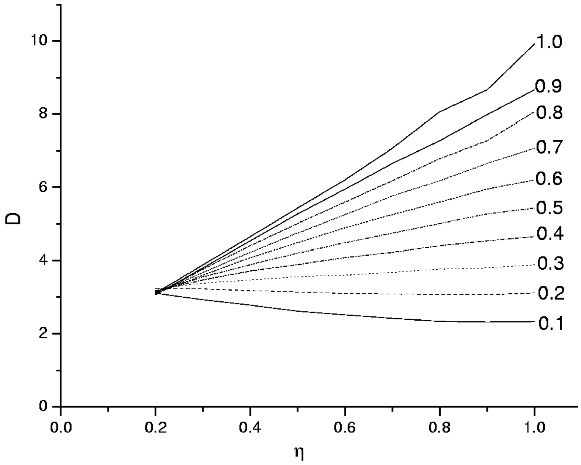

Fig. 1. The dependence $D(\eta)$ at various values of ζ for the Cloud C1 model at $\lambda = 0.7$ μm.

linear function at $\zeta = const$ and $\eta \geqslant 0.2$. However this is not the case, at $\eta < 0.2$ (not shown in Fig. 1). We see also some deviations from the linear dependence for the glory ($\zeta = 1, \eta \geqslant 0.9$) scattering region.

Note that calculations in Fig. 1 were performed for the Deirmendijan's cloud $C1$ model at the wavelength λ=0.7 μm [11]. Then it follows for the effective radius of droplets $a_{ef} \equiv \langle a^3 \rangle / \langle a^2 \rangle$=6 μm Here $\langle \cdots \rangle$ means averaging on the droplet size distribution $f(a) = Aa^6 e^{-9a/a_{ef}}$ (a is the radius of droplets, A is the normalization constant).

Performing the parameterisation of data presented in Fig. 1, we obtain

$$A \approx 3.944, \quad B \approx -2.5, \quad C \approx 10.664, \tag{15}$$

which differs from the isotropic scattering case, where $A \approx 1, B \approx 2, C \approx 4$. This underlines the importance of the parameter g (and the phase function) for the reflection function calculation. Melnikova et al. [7] give the following values for the parameters A, B, C as functions of g:

$$A = 1.12 + 1.9g, \quad B = 1.98 - 3g, \quad C = 3.51 + 4.8g. \tag{16}$$

They were obtained from the parameterizations of results of the solution of the Ambartsumian's nonlinear integral equation at single scattering albedo $\omega_0 = 1$ and the model phase function

$$p(\theta) = \frac{1 - g^2}{(1 + g^2 + 2g\cos\theta)^{3/2}}. \tag{17}$$

It follows from Eqs. (16) at $g = 0.85$:

$$A \approx 4, \quad B \approx -0.57, \quad C \approx 7.59, \tag{18}$$

which differ somewhat from our results obtained using $p(\theta)$, calculated using the electromagnetic wave theory [12] for distributions of water droplets.

We have finally,

$$R(\zeta, \eta, \varphi) = \frac{A + B(\zeta + \eta) + C\zeta\eta + F(\theta)}{4(\zeta + \eta)}, \tag{19}$$

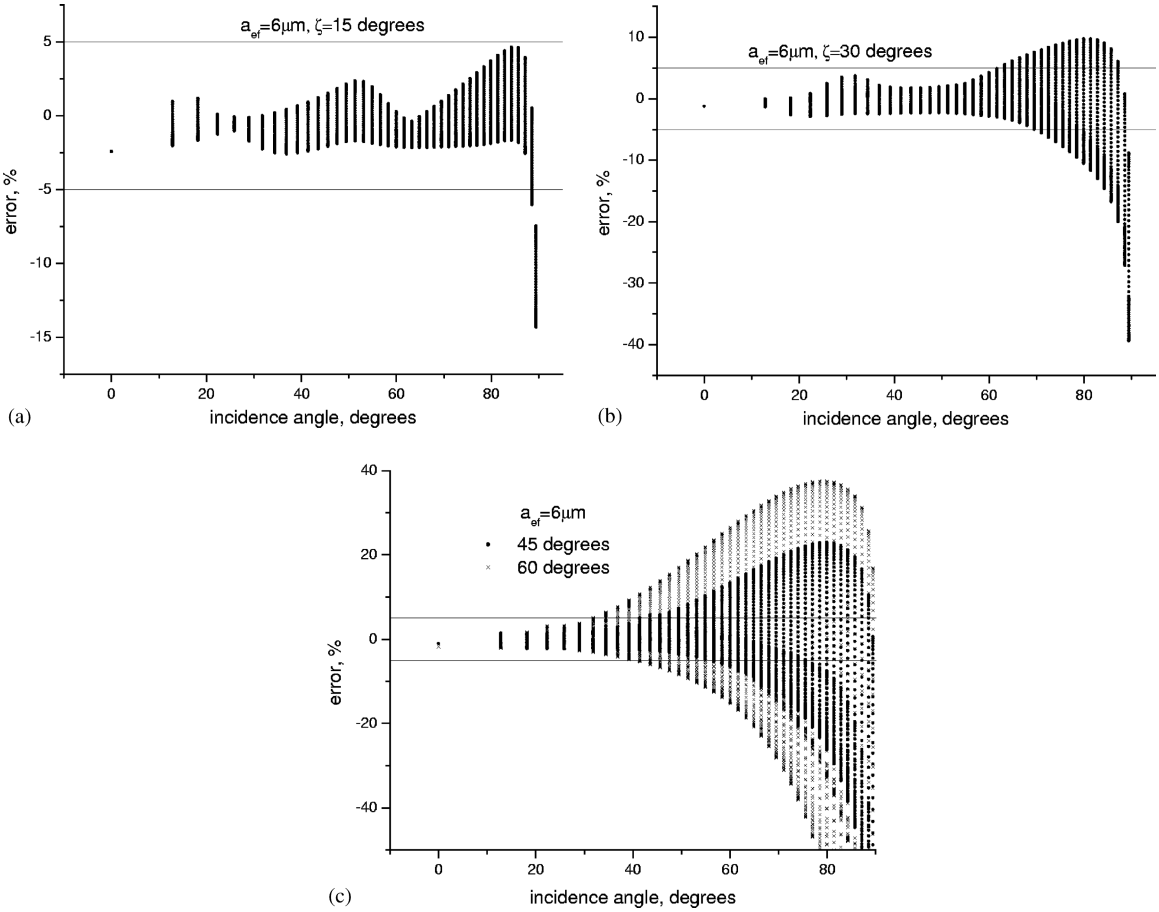

Fig. 2. The relative error of Eq. (19) as a function of the incidence angle at $\lambda = 0.65$ μm, $a_{\mathrm{ef}} = 6$ μm, $\vartheta_1 = 15°$ (a), $\vartheta_1 = 30°$ (b), $\vartheta_1 = 45°$ and 60° (c).

with A, B, C given in Eq. (15) and $F(\theta) = p(\theta) - \bar{p}(\theta)$. Eq. (19) reduces the calculation of $R(\zeta, \eta, \varphi)$ to the calculation of the phase function $p(\theta)$ of a given light scattering medium. Note that for particular cases (e.g., near-nadir observations) the function F in Eq. (19) can be dropped.

3. The accuracy of approximation

Let us study the accuracy of Eq. (19) as compared to exact radiative transfer calculations at the wavelength $\lambda = 0.65$ μm often used in cloud retrieval procedures [13]. The function $F(\theta)$ can be calculated performing the numerical integration in Eq. (5). We, however, applied a different approach here.

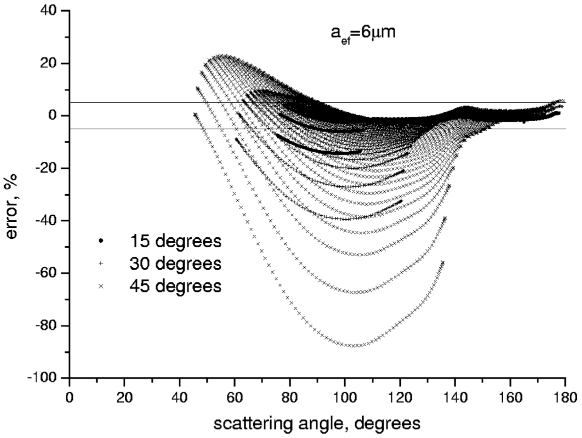

Fig. 3. The same as in Fig. 2 but as a function of the scattering angle θ.

In particular, we used the fact that the function $p(\theta)$ can be presented as

$$p(\theta) = \sum_{l=1}^{\infty} x_l P_l(\cos\theta), \tag{20}$$

where $P_l(\cos\theta)$ is the Legendre polynomial. We see that an infinite discrete set of numbers x_l completely determines the function $p(\theta)$. In practice, one can use a finite number M of terms in Eq. (20) ($M \approx 5x_{\mathrm{ef}}, x_{\mathrm{ef}} = 2\pi a_{\mathrm{ef}}/\lambda$). Using the addition theorem

$$P_l(\cos\theta) = P_l(\zeta)P_l(-\eta) + \sum_{s=1}^{l} h_l P_l^s(\zeta)P_l^s(\eta)\cos s\varphi, \tag{21}$$

where $h_l = 2(l-s)!/(l+s)!$ and $P_l^s(\zeta)$ are so-called associate Legendre polynomials, we obtain

$$\bar{p}(\theta) = \sum_{l=1}^{\infty} x_l P_l(\zeta)P_l(-\eta) \tag{22}$$

and, therefore,

$$F(\theta) = \sum_{l=1}^{\infty} x_l \left(P_l(\cos\theta) - (-1)^l P_l(\zeta)P_l(\eta)\right), \tag{23}$$

where we used the equality: $P_l(-\eta) = (-1)^l P_l(\eta)$.

The function $F(\theta)$ in Eq. (19) was calculated using Eq. (23) and pre-calculated set of coefficients x_l obtained with the Mie theory [12] for a given droplet size distribution.

Relative errors of Eq. (19) with account for Eqs. (15), (23) for spherical water droplets distribution at $a_{\mathrm{ef}} = 6$ μm and $\lambda = 0.65$ μm are presented in Figs. 2a–c. We see in particular that the absolute value of the relative error is below 2.5% for most of incidence angles and the observation angle 15°. The same is true for smaller observation angles. Therefore, Eq. (19) extends our earlier formulation [8] which was applicable only to the case of nadir measurements.

The errors are larger than 5% only for the case of incidence angles $\vartheta_0 \approx 90°$ and $\vartheta_1 \leqslant 15°$, which is not of particular interest as far as remote sensing of clouds is concerned.

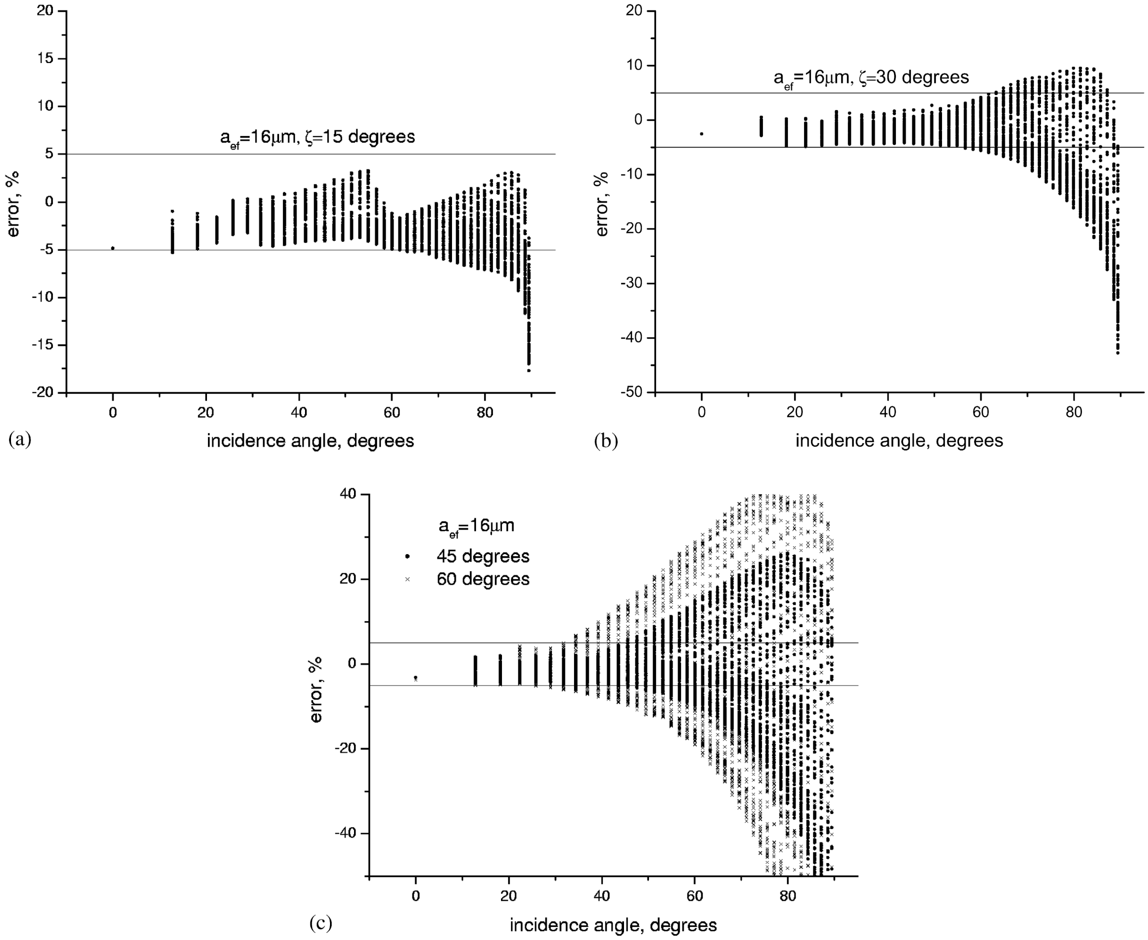

Fig. 4. The same as in Fig. 2 but at $a_{\mathrm{ef}} = 16$ μm.

Different points in the vertical direction in Figs. 2a–c represent the azimuth dependence of an error. In total, we have calculated 91 azimuth angles [$\varphi = 0°(2°)180°$] for each incidence angle. Therefore, Fig. 2 illustrates also the influence of azimuth harmonics on $R(\zeta, \eta, \varphi)$ (for twice and more times scattered light). Note that this influence is of increased importance in the rainbow ($\theta \approx 140°$) and glory ($\theta \approx 180°$) scattering regions.

As one might expect, errors increase with the observation angle. However, they are below 5% for the observation angles 0–30° (see Fig. 2b) and incidence angles smaller than 60°. They are below 7% at incidence angles smaller than 70°. This is also acceptable for most of remote sensing applications. Note that errors grow considerably at $\vartheta_0 \approx 90°$ and $\vartheta_1 = 30°$ (see Fig. 2b).

For observation angles 45° and 60° only the case of near-nadir illumination (0–30° within accuracy 5% and 0–40° within accuracy 10%) can be considered in the framework of the approximate formula introduced above (see Fig. 2c).

We present results of calculations given in Fig. 2 as functions of the scattering angle θ in Fig. 3 (except for the case $\vartheta_1 = 60°$). It follows that for large scattering angles ($\theta > 150°$) and observation

angles smaller than 45° the accuracy of Eq. (19) is better than 5% (except $\theta \approx 180°$, where it reaches 7%). The same is true till $\theta \approx 130°$ at $\vartheta_1 < 30°$.

To check the applicability of Eq. (19) to clouds with larger droplets, we have repeated the same calculations at $a_{\mathrm{ef}} = 16\ \mu\mathrm{m}$ and give them in Fig. 4. The error is somewhat larger in this case. However, it is below 7% at observation angles below 30° and incidence angles smaller than 70°. This also indicates the importance of the scattering angle θ, given in Eq. (6) for the accuracy of Eq. (19).

4. Conclusion

We have proposed approximate Eq. (19) to calculate the reflection function of semi-infinite cloudy media. This equation is similar to that presented by Melnikova et al. [7] for the Henyey–Greenstein phase function. The accuracy of approximate solution obtained was studied in detail for the water droplets distributions $f(a) = Aa^6 \exp(-9a/a_{\mathrm{ef}})$ at $\lambda = 0.65\ \mu\mathrm{m}$, $a_{\mathrm{ef}} = 6$ and $16\ \mu\mathrm{m}$. The azimuth was varied in the range 0–180° with step 2° for incidence angles in the range 0–90° and observation angles 0–60°.

It was found that the error of Eq. (19) is below 7% for incidence angles smaller than 70° and observation angles smaller than $30° (a_{\mathrm{ef}} = 6\text{–}16\ \mu\mathrm{m})$. For most part of this range the error is below 5%. Therefore, Eq. (19) can be used in a number of atmospheric optics applications and in particular for the satellite cloud retrieval procedures simplification [13,14].

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